On the Theory of the Magnetic Resistivity near a Ferromagnetic Phase Transition

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Perturbation theory is applied to the evaluation of the magnetic resistivity of a magnetic metal near a ferromagnetic phase transition. The method starts from first principles and uses diagrammatic expansions together with the theory of spin cumulant averages. Emphasis is put on some critical hypothesis regarding the behaviour of the coupled electron-spin system. A generalization of the de-Gennes-Friedel expressions for the static conductivity is derived and arguments are advanced regarding its energy like critical behaviour.

A short discussion is also given for the frequency dependent conductivity and it is shown that it deviates from Drude's law.

1. Introduction

The critical behaviour of transport coefficients in magnetic metals or alloys has been the subject of intense research, both experimental and theoretical. Recent techniques [1, 2, and references therein] have made available precise determination of the temperature derivative of transport coefficients, specially near a magnetic transition. The theoretical interpretation of these results is, however, still controversial [3], due basically to two reasons:

(i) expressions for the coefficients are obtained through approximate theories (e.g., Boltzmann equation), together with not completely justified assumptions (e.g., existence of a relaxation time, use of Born approximation, etc.).

(ii) molecular field or random phase approximations are then brought in to discuss critical behaviour of the spin cumulants entering the formulae. This second type of approximations can, of course, be improved by the use of scaling expressions and the correct exponents, when they are known. However, one often requires knowledge of the critical behaviour of, e.g., the two-spin correlation for all wave vectors. The first type of approximation is, in our opinion, more fundamental and more difficult to remedy, since the derivation of useful expressions is full of approximations whose irrelevance for critical behaviour is not obvious and

seldom discussed. As an example, the resistivity of magnetic metals is largely discussed in terms of the de Gennes-Friedel expression [4], [5], which assumes the existence of a relaxation time, the validity of Born approximation and elastic collisions which, of course, may be not valid near a critical point [6].

It is the purpose of this paper to clarify and generalize the derivation of expressions for the kinetic coefficients. We shall start from first principles (Sect. 2) and make use of standard techniques in many-body theory (Sects. 3 and 4). We shall make clear which approximations are considered and discuss their relevance for critical behaviour of magnetic phenomena. In this work emphasis will be put on the generalized, frequency-dependent conductivity of ferromagnetic metals, assuming localized spins which scatter conduction electrons through spin-spin interaction. The method is however quite general and can be used to discuss other types of magnetic ordering as well as other transport coefficients.

2. Basic Theory

We start from Kubo's expression of the macroscopic conductivity tensor [7] for an homogeneous crystal:

$$\sigma_{\mu\nu}(\omega) = \frac{e^2}{V} \sum_{\substack{\varkappa, \varkappa' \\ \lambda, \lambda'}} v_\mu v_\nu' \tag{1}$$

$$\cdot \left[\frac{\mathscr{D}_{\lambda\lambda'}(\bar{\varkappa}, \bar{\varkappa}', \nu_n) - \mathscr{D}_{\lambda\lambda'}(\bar{\varkappa}, \bar{\varkappa}', 0)}{\hbar \nu_n} \right]_{i\nu_n = \omega + i\eta},$$

where V is the volume of the crystal, v_μ denotes the cartesian component of the electron velocity $\bar{v}(\bar{\varkappa}) = \partial \varepsilon / \partial \bar{\varkappa}$ and $\mathscr{D}_{\lambda\lambda'}(\bar{\varkappa}, \bar{\varkappa}', \nu_n)$ is essentially the Fourier transform of the density-density thermal Green's function [8]:

$$\mathscr{D}_{\lambda\lambda'}(\bar{\varkappa}, \bar{\varkappa}', \nu_n) = - \int_0^{\beta\hbar} \mathrm{d}(\tau - \tau') e^{i\nu_n(\tau - \tau')} \langle T c_{\bar{\varkappa}\lambda}^+(\tau) c_{\bar{\varkappa}\lambda}(\tau) c_{\bar{\varkappa}'\lambda'}^+(\tau') c_{\bar{\varkappa}'\lambda'}(\tau') \rangle. \tag{2}$$

We now assume that the Hamiltonian can be divided into three parts:

$$\mathscr{H} = \mathscr{H}_{\mathrm{el}} + \mathscr{H}_{\mathrm{sp}} + \mathscr{H}_{\mathrm{int}},$$

where $\mathscr{H}_{\mathrm{el}}$ denotes the contribution from the conduction electrons:

$$\mathscr{H}_{\mathrm{el}} = \sum_{\bar{\varkappa}, \lambda} \varepsilon(\bar{\varkappa}) c_{\bar{\varkappa}\lambda}^+ c_{\bar{\varkappa}\lambda}.$$

For the moment, we do not need to specify $\mathscr{H}_{\mathrm{sp}}$, representing the contribution from localized spins (it may be even absent as, for instance is usually assumed in spin glasses). Finally, the last term represents the usual s–d interaction:

$$\mathscr{H}_{\mathrm{int}}$$
$$= - \frac{1}{V} \sum_{\substack{\bar{\varkappa}, \bar{\varkappa}' \\ \alpha, \alpha'}} J(\bar{\varkappa} - \bar{\varkappa}') \, \bar{\sigma}_{\alpha\alpha'} \cdot \bar{s}(\bar{\varkappa} - \bar{\varkappa}') c_{\bar{\varkappa}\alpha}^+ c_{\bar{\varkappa}'\alpha'}.$$

This is responsible for an indirect coupling between the spins (leading, to lowest order, to a RKKY long-range oscillating interaction) and, of course, it also causes electron scattering, being therefore the origin of the magnetic anomalies in transport coefficients. We treat this term in perturbation theory, using Feynman diagrams (Figure 1).

$$\mathscr{D}_{\lambda\lambda'}(\bar{\varkappa}\tau, \bar{\varkappa}'\tau')$$

$$\mathscr{G}_\lambda^0(\bar{\varkappa}, \tau - \tau')$$

$$- \frac{1}{V} J(\bar{\varkappa} - \bar{\varkappa}') \, \bar{\sigma}_{\alpha\alpha'} \cdot s(\bar{\varkappa} - \bar{\varkappa}', \tau) c_{\bar{\varkappa}\alpha}^+(\tau) c_{\bar{\varkappa}'\alpha'} c(\tau)$$
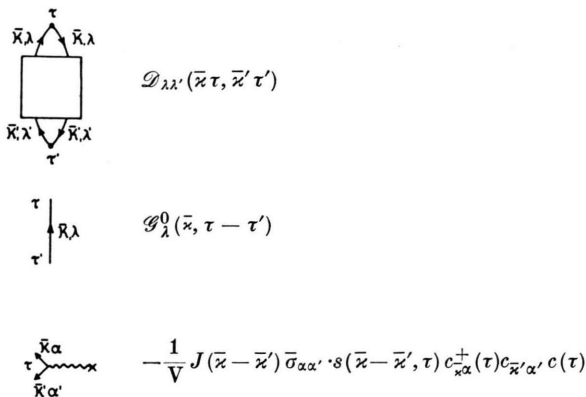
Fig. 1. Diagrammatic representation of the two-particle Green's function, the single-particle Green's function and basic interaction vertex.

Then, only linked diagrams contribute to (2), provided we pay attention to the following features:

(a) Diagrams which split into two parts, one attached to the bottom of $\mathscr{D}$ and the other to the upper part of $\mathscr{D}$, represent the perturbation series for the electron density and give no contribution to (1). This is obviously the statement that only current fluctuations enter in (1).

(b) Spin averages must be expressed in terms of time-ordered spin cumulants [9]; only in this way will unlinked terms cancel in the numerator and denominator of the perturbation expression for D. We shall henceforth represent the n-th order spin cumulant by a point with n wiggly lines attached. Notice that the time-ordering for the spin operation is defined as for bosons.

In Fig. 2 we represent all linked diagrams, for D up to 2nd order.

We are now in a position which permits several simplifying remarks:

(i) Many of the terms for the perturbation series for D can be formally summed up immediately, since they just amount to the renormalization of the electron-propagator to its exact value (this is the case, e.g. of the diagrams of 0th and 1st order and diagrams (a) to (g) in 2nd order).

(ii) Diagrams which connect the upper part and the bottom of D do not contribute to (1). This is clearly seen for diagrams (h) in Fig. 2, but can also
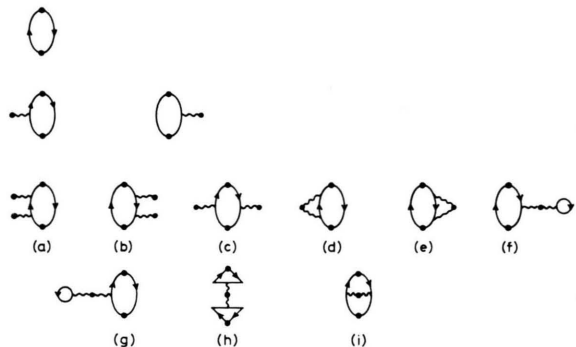

Fig. 2. All linked diagrams up to 2nd order.

be shown to hold in higher orders [10], provided $\varepsilon(\bar{\varkappa})$ and the spin correlations are invariant under momentum inversion.

(iii) We call reducible diagram that which has parts attached to the rest of the diagram, only through a dot (e.g., diagrams (f) and (g) in Figure 2). The dot is then an articulation point. These diagrams may, too, be summed up [9], since they just renormalize the time dependence of the spin cumulants in the interaction picture, to the Heisenberg one (Figure 3). Thus we eliminate all reducible diagrams and will not accept henceforth articulation points (we still keep the representation by a dot of the now renormalized spin cumulant vertex).
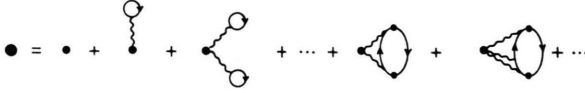

Fig. 3. Renormalization of spin cumulant vertex.

This last remark is very important: it states that any singularity presented in the spin cumulants is due to the full Hamiltonian and not just $\mathscr{H}_{sp}$ (which may even be absent as is accepted in some spin glasses).

We shall now introduce our first grand assumption: we neglect all spin cumulants of 3rd or higher order, on the grounds that these have less singular behaviour than the 1st (order parameter) or 2nd (susceptibility-like) ones. This is a very crucial hypothesis whose validity must be tested for each spin Hamiltonian.

3. Electron Propagator and Spin Correlations

With the usual definition of the single-particle Green's function:

$$\mathscr{G}_\lambda(\bar{\varkappa}, \tau - \tau') = -\langle T c_\lambda(\bar{\varkappa}, \tau) c_\lambda^+(\bar{\varkappa}, \tau')\rangle$$

Fig. 4. Dyson's equation for the electron propagator and lowest order contribution to the self-energy.

and its Fourier transform

$$\mathscr{G}_\lambda(\bar{\varkappa}, \omega_n) = \int_0^{\beta\hbar} d(\tau - \tau') e^{i\omega_n(\tau-\tau')} \mathscr{G}_\lambda(\bar{\varkappa}, \tau - \tau'),$$

(where $\omega_n = (n + \tfrac{1}{2}) 2\pi/\beta\hbar$) we easily set up a Dyson equation (Figure 4)

$$\mathscr{G}_\lambda(\bar{\varkappa}, \omega_n) = \mathscr{G}_\lambda^0(\bar{\varkappa}, \omega_n) + \mathscr{G}_\lambda^0(\bar{\varkappa}, \omega_n) \Sigma_\lambda(\bar{\varkappa}, \omega_n) \mathscr{G}_\lambda(\bar{\varkappa}, \omega_n).$$

The proper self-energy is given, under the first assumption, by:

$$\hbar \Sigma_\lambda(\bar{\varkappa}, \omega_n) = -\frac{J(0)}{V} \qquad (4)$$
$$\cdot \langle \bar{\sigma}_{\lambda\lambda} \cdot \bar{s}(0)\rangle + \hbar \Sigma_\lambda^{(2)}(\bar{\varkappa}, \omega_n).$$

Here, the first term just shifts the unperturbed electron energy and is proportional to the (full) magnetization. We shall henceforth assume that this term has been included in the electron Hamiltonian:

$$\varepsilon(\bar{\varkappa}) \to \varepsilon_\lambda(\bar{\varkappa}) = \varepsilon(\bar{\varkappa}) - \frac{J(0)}{V} \langle \bar{\sigma}_{\lambda\lambda} \cdot \bar{s}(0)\rangle.$$

The second term in (4) arises from a double scattering of the electron by the spin systems and is therefore proportional to the 2nd order spin cumulant:

$$\hbar \Sigma_\lambda^{(2)}(\bar{\varkappa}, \omega_n) = \frac{1}{V} \sum_{\bar{\varkappa}', \lambda'} |J(\bar{\varkappa} - \bar{\varkappa}')|^2 \frac{1}{\beta\hbar^2} \sum_{\omega_{n'}} \mathscr{G}_\lambda^0(\bar{\varkappa}', \omega_{n'}) \Gamma_{\lambda\lambda'}(\bar{\varkappa} - \bar{\varkappa}', \omega_n - \omega_{n'}), \qquad (5)$$

where

$$\Gamma_{\lambda\lambda'}(\bar{q}, \nu_n) = \frac{1}{V} \int_0^{\beta\hbar} d(\tau - \tau') e^{i\nu_n(\tau-\tau')} \langle T \bar{\sigma}_{\lambda\lambda'} \cdot \bar{s}(\bar{q}, \tau) \bar{\sigma}_{\lambda'\lambda} \cdot \bar{s}(-\bar{q}, \tau')\rangle_c. \qquad (6)$$

This is linked with the real time spin correlations through the spectral representation:

$$\Gamma_{\lambda\lambda'}(\bar{q}, \nu_n) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\omega \frac{\Gamma_{\lambda\lambda'}(\bar{q}, \omega)}{i\nu_n - \omega}, \qquad (7)$$

$$\Gamma_{\lambda\lambda'}(\bar{q}, \omega) = -(1 - e^{-\beta\hbar\omega}) S_{\lambda\lambda'}(\bar{q}, \omega), \qquad (8)$$

$$S_{\lambda\lambda'}(\bar{q},\omega) = \frac{1}{V} \int\limits_{-\infty}^{+\infty} \mathrm{d}t\, e^{-i\omega t} \langle \overline{\sigma}_{\lambda\lambda'} \cdot \bar{s}(\bar{q},t)\, \overline{\sigma}_{\lambda'\lambda} \cdot \bar{s}(-\bar{q},0)\rangle_c\,. \tag{9}$$

Using the spectral representation for the electron propagator (g) and the self-energy (σ), we readily obtain:

$$\hbar\,\sigma_\lambda(\bar{\varkappa},\omega) = \frac{1}{\hbar}\sum_{\lambda'}\int\frac{\mathrm{d}\bar{\varkappa}'}{(2\pi)^3}\,|J(\bar{\varkappa}-\bar{\varkappa}')|^2\int\limits_{-\infty}^{+\infty}\frac{\mathrm{d}\omega'}{2\pi}\,g_{\lambda'}(\bar{\varkappa}',\omega')\,S_{\lambda\lambda'}(\bar{\varkappa}-\bar{\varkappa}',\omega-\omega')\,\frac{1+e^{-\beta\hbar\omega}}{1+e^{-\beta\hbar\omega'}}\,. \tag{10}$$

We shall accept that the electron-spin interaction is a small perturbation. More precisely, we assume that $J(0)(N/V)S \ll \mu$, where N is the number of spins (each of magnitude S) and μ is the Fermi energy. Moreover, one expects the electron-spin coupling to be mainly short-range: hence, $J(\bar{\varkappa}) \approx J(0)$.

We may further approximate (10) along the following lines:

(a) Near a magnetic transition, the spin correlation function $S_{\lambda\lambda'}(\bar{q},\omega)$ has a singular part which is crucial for the critical behaviour, and a regular part. This we leave aside, since it amounts to a small non critical correction, on equal footing with phonon scattering, and which can be absorbed in the expression for the Green's function $\mathscr{G}_\lambda^0(\bar{\varkappa},\omega_n)$. Therefore, we are ignoring diagrams expressing the overlap of the critical spin mechanism with other types of scattering — this is our second grand assumption.

(b) Near a ferromagnetic transition, restricted dynamic scaling [11] assumes.

$$S_{\lambda\lambda'}(\bar{q},\omega) = \frac{2\pi}{\omega_c(k,q)}\,S_{\lambda\lambda'}(q)\,F_{\lambda\lambda'}\!\left(\frac{q}{k},\frac{\omega}{\omega_c}\right), \tag{11}$$

where k is the inverse of the correlation length $(k \sim t^\nu)$ and $S_{\lambda\lambda'}(q)$ is the static correlation function:

$$S_{\lambda\lambda'}(q) = \frac{1}{V}\langle\overline{\sigma}_{\lambda\lambda'}\cdot\bar{s}(\bar{q})\,\overline{\sigma}_{\lambda'\lambda}\cdot\bar{s}(-\bar{q})\rangle\,. \tag{12}$$

Also, $\omega_c(k,q)$ is the inverse of a characteristic relaxation time, with a scaling behaviour:

$$\omega_c(k,q) = q^z\,G_{\lambda\lambda'}(q/k) \tag{13}$$

where the exponent $z \approx 3$.

Finally, the last term in (11) is the shape function $F_{\lambda\lambda'}(q/k,\omega/\omega_c)$ which, by definition, is normalized to unity:

$$\int\limits_{-\infty}^{+\infty} F_{\lambda\lambda'}\!\left(\frac{q}{k},k\right)\mathrm{d}k = 1\,. \tag{14}$$

(c) Accepting the dynamic scaling it is obvious that the frequency integral in (10) is dominated by the region $|\omega-\omega'|\sim\omega_c(|\bar{\varkappa}-\bar{\varkappa}'|)$, which is rather small for $|\bar{\varkappa}-\bar{\varkappa}'|\lesssim k$; on the other hand, for $|\bar{\varkappa}-\bar{\varkappa}'|>k$, the correlation function decreases very rapidly. Therefore, we get:

$$\hbar\,\sigma_\lambda(\bar{\varkappa},\omega) \approx \sum_{\lambda'}\int\frac{\mathrm{d}\bar{\varkappa}'}{(2\pi)^3}\,\frac{|J(\bar{\varkappa}-\bar{\varkappa}')|^2}{\hbar}\,\cdot g_{\lambda'}(\bar{\varkappa}',\omega)\,S_{\lambda\lambda'}(\bar{\varkappa}-\bar{\varkappa}')\,.$$

(d) We may further approximate this expression through the substitution of the full electron propagator, $g_\lambda(\bar{\varkappa},\omega)$ by the zero-order one, $g_\lambda^0(\bar{\varkappa},\omega)$. We must, however, note that this should not be taken as the free particle propagator:

$$g_\lambda^{\mathrm{free}}(\bar{\varkappa},\omega) = 2\pi\,\delta(\omega-\hbar^{-1}(\varepsilon_\lambda(\bar{\varkappa})-\mu))\,.$$

Rather, $g_\lambda^0(\bar{\varkappa},\omega)$ is the propagator associated with electron motion in the absence of critical magnetic scattering; it thus includes other types of electron scattering (e.g. phonons, non magnetic impurities or the regular part of magnetic fluctuations). This is a rather crucial remark which needs additional consideration. If the free particle propagator was inserted, we would eventually obtain the de-Gennes-Friedel expression [4], whereby electron scattering takes place in the Fermi surface. It thus leads to wrong critical behaviour [5], due to the two-dimensional integration [10]. On the contrary, proceeding as we stated, we still keep the three dimensional integration. This can be traced back to the fact that in between two consecutive collisions with the spin system, the electron does not propagate freely, and therefore on the same energy shell. Instead, the electron may exchange energy with other scattering sources and it thus leads to a space-dependent factor which works as a cutoff for the scattering by two-spins wide apart [5].

Under these assumptions we finally obtain:

$$\hbar\,\sigma_\lambda(\bar{\varkappa},\omega) \cong \sum_{\lambda'}\int\frac{\mathrm{d}\bar{\varkappa}'}{(2\pi)^3}\,\frac{|J(\bar{\varkappa}-\bar{\varkappa}')|^2}{\hbar^2}\,\cdot g_{\lambda'}^0(\bar{\varkappa}',\omega)\,S_{\lambda\lambda'}(\bar{\varkappa}-\bar{\varkappa}')\,. \tag{15}$$

We conclude this rather long section by noting that the scaling assumption leads naturally to an approximate expression for the thermal-spin Green's function (see (7)):

$$\Gamma_{\lambda\lambda'}(\bar{q}, \nu_n) \approx \beta\hbar\, S_{\lambda\lambda'}(q)\, \delta_{\nu_n, 0}. \qquad (16)$$

Therefore, the spin systems act as a source of instantaneous interactions between the electrons.

4. The Two-Particle Green's Function

We may now return to the evaluation of the electron-hole Green's function $\mathscr{D}_{\lambda\lambda'}(\bar{\varkappa}, \bar{\varkappa}', \nu_n)$. Under the two basic assumptions previously discussed, this is just the sum of ladder diagrams (Fig. 5) to lowest order

$$\mathscr{D}^{(0)}_{\lambda\lambda'}(\bar{\varkappa}, \bar{\varkappa}', \nu_n) = \delta_{\bar{\varkappa}\bar{\varkappa}'}\, \delta_{\lambda\lambda'}$$
$$\cdot \frac{1}{\beta\hbar} \sum_{\omega_n} \Omega_\lambda(\bar{\varkappa}, \omega_n, \nu_n), \qquad (17)$$
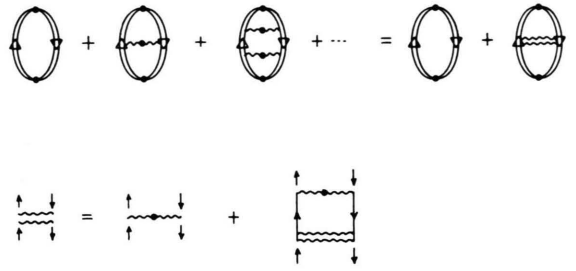

Fig. 5. Ladder series for two-particle Green's function and effective electron-electron interaction.

where

$$\Omega_\lambda(\bar{\varkappa}, \omega_n, \nu_n) = \mathscr{G}_\lambda(\bar{\varkappa}, \omega_n)$$
$$\cdot \mathscr{G}_\lambda(\bar{\varkappa}, \omega_n + \nu_n). \qquad (18)$$

The ladder series can conveniently be summed up through the definition of an effective electron-electron interaction (Figure 5):

$$v^*_{\lambda\lambda'}(\bar{\varkappa}, \bar{\varkappa}', \omega_n, \nu_n) = v_{\lambda\lambda'}(\bar{\varkappa} - \bar{\varkappa}') + \frac{1}{V} \sum_{\lambda_1, \bar{\varkappa}'_1} v_{\lambda\lambda_1}(\bar{\varkappa} - \bar{\varkappa}_1)\, \Omega_{\lambda_1}(\bar{\varkappa}_1, \omega_n, \nu_n)\, v^*_{\lambda_1\lambda'}(\bar{\varkappa}, \bar{\varkappa}', \omega_n, \nu_n), \qquad (19)$$

where
$$v_{\lambda\lambda'}(\bar{q}) = \frac{|J(\bar{q})|^2}{\hbar^2}\, S_{\lambda\lambda'}(\bar{q}). \qquad (20)$$

Thus

$$\mathscr{D}_{\lambda\lambda'}(\bar{\varkappa}, \bar{\varkappa}', \nu_n) = \frac{1}{\beta\hbar} \sum_{\omega_n} \Omega_\lambda(\bar{\varkappa}, \omega_n, \nu_n) \left\{ \delta_{\lambda\lambda'}\, \delta_{\bar{\varkappa}\bar{\varkappa}'} + \frac{v^*_{\lambda\lambda'}}{V}(\bar{\varkappa}, \bar{\varkappa}', \omega_n, \nu_n)\, \Omega_{\lambda'}(\bar{\varkappa}', \omega_n, \nu_n) \right\}. \qquad (21)$$

Having in mind Eq. (1), we put

$$\sum_{\bar{\varkappa}'\lambda'} \mathscr{D}_{\lambda\lambda'}(\bar{\varkappa}, \bar{\varkappa}', \nu_n)\, \bar{v}(\bar{\varkappa}') \equiv \frac{1}{\beta\hbar} \sum_{\omega_n} \bar{\pi}_\lambda(\bar{\varkappa}, \omega_n, \nu_n). \qquad (22)$$

Then, from eqs. (19) and (21), we get:

$$\bar{\pi}_\lambda(\bar{\varkappa}, \omega_n, \nu_n) = \Omega_\lambda(\bar{\varkappa}, \omega_n, \nu_n) \left\{ \bar{v}(\bar{\varkappa}) + \frac{1}{V} \sum_{\bar{\varkappa}', \lambda'} v_{\lambda\lambda'}(\bar{\varkappa} - \bar{\varkappa}')\, \bar{\pi}_{\lambda'}(\bar{\varkappa}', \omega_n, \nu_n) \right\}. \qquad (23)$$

We now assume that near a ferromagnetic transition, $v_{\lambda\lambda'}(\bar{q})$ is spherically symmetric and is appreciable only for $|\bar{q}| \lesssim k$. Thus, we may solve (23) by projecting $\bar{\pi}_\lambda$ along $\bar{v}$, yielding:

$$\bar{\pi}_\lambda(\bar{\varkappa}, \omega_n, \nu_n) = \Omega_\lambda(\bar{\varkappa}, \omega_n, \nu_n)\, \Lambda_\lambda(\bar{\varkappa}, \omega_n, \nu_n)\, \bar{v}(\bar{\varkappa}),$$

where

$$\Lambda_\lambda(\bar{\varkappa}, \omega_n, \nu_n) = 1 + \frac{1}{V} \sum_{\bar{\varkappa}'\lambda'} v_{\lambda\lambda'}(\bar{\varkappa} - \bar{\varkappa}')\, \Omega_{\lambda'}(\bar{\varkappa}', \omega_n, \nu_n)\, \frac{\bar{v} \cdot \bar{v}'}{|\bar{v}|^2}\, \Lambda_{\lambda'}(\bar{\varkappa}', \omega_n, \nu_n) \approx 1/(1 - \Lambda_\lambda(\bar{\varkappa}, \omega_n, \nu_n)$$

with

$$A_\lambda(\bar{\varkappa}, \omega_n, \nu_n) = \sum_{\lambda'} \int \frac{d\varkappa'}{(2\pi)^3} v_{\lambda\lambda'}(\bar{\varkappa} - \bar{\varkappa}')\, \Omega_{\lambda'}(\bar{\varkappa}', \omega_n, \nu_n)\, \frac{\bar{v} \cdot \bar{v}'}{|\bar{v}|^2}. \qquad (24)$$

Thus
$$\bar{\pi}_\lambda(\bar{\varkappa}, \omega_n, \nu_n) = \frac{\Omega_\lambda(\bar{\varkappa}, \omega_n, \nu_n)}{1 - A_\lambda(\bar{\varkappa}, \omega_n, \nu_n)}\, \bar{v}(\bar{\varkappa})\,. \tag{25}$$

The rest of the calculation now proceeds along well known paths and it is summarized in the Appendix. Here, we just quote the final result, assuming spherical symmetry which simplifies considerably the discussion:

$$\sigma(\omega) = \frac{e^2}{3\hbar} \sum_\lambda \int \frac{d\omega'}{2\pi} \frac{n(\omega' - \omega) - n(\omega')}{\omega}$$
$$\cdot \int \frac{d\bar{\varkappa}}{(2\pi)^3} \frac{|\bar{v}(\bar{\varkappa})|^2}{1 - \eta_\lambda(\bar{\varkappa}, \omega', \omega)} \frac{\frac{1}{2}[g_\lambda(\bar{\varkappa}, \omega') + g_\lambda(\bar{\varkappa}, \omega' - \omega)]}{-i\omega + \gamma_\lambda(\bar{\varkappa}, \omega', \omega)}\,. \tag{26}$$

Here, $n(\omega) = [1 + \exp(\beta\hbar\omega)]^{-1}$ and

$$\gamma_\lambda(\bar{\varkappa}, \omega', \omega) = \tfrac{1}{2}\sigma_\lambda(\bar{\varkappa}, \omega') + \tfrac{1}{2}\sigma_\lambda(\bar{\varkappa}, \omega' - \omega) + \sigma_0\,, \tag{27}$$

$$\eta_\lambda(\bar{\varkappa}, \omega', \omega) = \sum_{\lambda'} \int \frac{d\bar{\varkappa}'}{(2\pi)^3} \frac{|J(\bar{\varkappa} - \bar{\varkappa}')|^2}{\hbar^2} \frac{\bar{v}(\bar{\varkappa}) \cdot \bar{v}(\bar{\varkappa}')}{|\bar{v}(\bar{\varkappa})|^2}$$
$$\cdot S_{\lambda\lambda'}(\bar{\varkappa} - \bar{\varkappa}') \frac{\frac{1}{2}[g_{\lambda'}(\bar{\varkappa}', \omega') + g_{\lambda'}(\bar{\varkappa}', \omega' - \omega)]}{-i\omega + \gamma_{\lambda'}(\bar{\varkappa}', \omega', \omega)}\,, \tag{28}$$

where $\sigma_\lambda(\bar{\varkappa}, \omega)$, given by (15), and σ_0, are, respectively, the contributions to the self-energy by the critical spin fluctuations and by the other scattering mechanisms (e. g., phonons). If we denote by $\sigma_\lambda^t(\bar{\varkappa}, \omega)$ the total contribution,

$$\sigma_\lambda^t(\bar{\varkappa}, \omega) = \sigma_\lambda(\bar{\varkappa}, \omega) + \sigma_0$$

then

$$g_\lambda(\bar{\varkappa}, \omega) = \frac{\sigma_\lambda^t(\bar{\varkappa}, \omega)}{(\omega - \omega_\lambda(\bar{\varkappa}))^2 + \left(\dfrac{\sigma_\lambda^t(\bar{\varkappa}, \omega)}{2}\right)^2}\,, \tag{29}$$

where we have neglected the real part of the self-energy. Obviously, the assumed smallness of $\sigma_\lambda^t(\bar{\varkappa}, \omega)$ means that $g_\lambda(\bar{\varkappa}, \omega)$ is approximately Lorentzian.

5. Discussion

We first consider the static conductivity. Putting $\omega = 0$ in (26) and assuming, as usual, that $K_B T \ll \mu$, we get:

$$\sigma(0) = \frac{e^2}{3\hbar} \sum_\lambda \int \frac{d\bar{\varkappa}}{(2\pi)^3} \frac{|\bar{v}(\bar{\varkappa})|^2}{1 - \eta_\lambda(\bar{\varkappa}, 0, 0)}$$
$$\cdot \frac{g_\lambda(\bar{\varkappa}, 0)/2\pi}{\gamma_\lambda(\bar{\varkappa}, 0, 0)}\,.$$

The Lorentzian approximation to g_λ allows the integration to be performed provided the rest of the integrand varies smoothly on an energy shell, of width $\hbar\sigma_\lambda^t(\bar{\varkappa}_F, 0)$ around the Fermi surface. If we call

$$f_\lambda \equiv \int d\bar{\varkappa}/(2\pi)^3 \tfrac{1}{2} m |\bar{v}(\bar{\varkappa})|^2$$
$$\cdot \delta(\mu - \varepsilon_\lambda(\bar{\varkappa})) \approx \tfrac{3}{4} n \tag{30}$$

(the estimate was derived for a free-electron gas whose density is n), we get

$$\sigma(0) \approx \frac{n e^2}{2m} \tag{31}$$
$$\cdot \sum_\lambda \frac{1}{\gamma_\lambda(\bar{\varkappa}_F, 0, 0)[1 - \eta_\lambda(\bar{\varkappa}_F, 0, 0)]}\,.$$

In the paramagnetic region, we obtain

$$\sigma(0) = n e^2 \tau/m\,,$$

where, using (27), (28) and (15), we have

$$\frac{1}{\tau} = \sigma_0 + \int \frac{d\bar{\varkappa}'}{(2\pi)^3} \frac{|J(\bar{\varkappa}_F - \bar{\varkappa}')|^2}{\hbar^2}$$
$$\cdot \langle \bar{s}(\bar{\varkappa}_F - \bar{\varkappa}') \cdot \bar{s}(\bar{\varkappa}' - \bar{\varkappa}_F) \rangle$$
$$\cdot \left[1 - \frac{\bar{v}(\bar{\varkappa}_F) \cdot \bar{v}(\bar{\varkappa}')}{|\bar{v}(\bar{\varkappa}_F)|^2}\right] g(\bar{\varkappa}', 0)\,. \tag{32}$$

The magnetic contribution to the resistivity is very similar to the de-Gennes-Friedel expression [4] and, indeed reduces to it if we substitute the electron propagator by the free-electron one. This however, is wrong [5], as we have discussed before. Of course, we may approximate g by g^0, but this maintains

the three-dimensional integration on a region of width $\hbar\sigma_0$ around the Fermi surface. Therefore, the long-range spin correlation triggered by the magnetic transition is really cut-off at distances greater than the non-magnetic mean free path. Hence, the resistivity must pick-up the short range correlations, thus behaving like the magnetic energy [6]. Consider, now, the ferromagnetic regime and assume $\sigma_0 \gg \sigma_\lambda$. Then, the electron propagator g_λ may be substituted, in (15) and (28), by the approximately spin-independent zeroth order propagator, g^0. In (31), we now have:

$$\gamma_\lambda (1 - \eta_\lambda) \cong \sigma_0 + \int \frac{d\bar{\varkappa}'}{(2\pi)^3} \frac{|J(\bar{\varkappa}_F - \bar{\varkappa}')|^2}{\hbar^2} \left[1 - \frac{\bar{v}(\bar{\varkappa}_F) \cdot \bar{v}(\bar{\varkappa}')}{|\bar{v}(\bar{\varkappa}_F)|^2} \right]$$
$$\cdot \left[\langle \bar{s}(\bar{\varkappa}_F - \bar{\varkappa}') \cdot \bar{s}(\bar{\varkappa}' - \bar{\varkappa}_F) \rangle_c - \frac{N}{V} \bar{\sigma}_{\lambda\lambda} \cdot \langle \bar{s} \rangle \right] g^0(\bar{\varkappa}', 0) .$$

Therefore, in the vicinity of the Curie temperature the resistivity should be of the form

$$\varrho = \varrho_m - \frac{b}{\varrho_m} \langle \bar{s} \rangle^2 \left(\frac{N}{V} \right)^2 ,$$

where ϱ_m is given by [32], with the substitution of the spin correlation by the second-order cumulant, and b is a positive constant.

This behaviour appears to be observed experimentally in some rare-earth metals (J. Bessa Sousa, private communication).

The frequency-dependent conductivity is much more difficult to analize. We here restrict ourselves to the paramagnetic regime and to the region $\hbar\omega \ll \mu$. From (26) we then get:

$$\sigma(\omega) \cong \frac{n e^2}{m} \frac{1}{-i\omega + \gamma^t(\bar{\varkappa}_F, 0, \omega)} , \tag{33}$$

where

$$\gamma^t(\bar{\varkappa}_F, 0, \omega) = \sigma_0 + \int \frac{d\bar{\varkappa}'}{(2\pi)^3} \frac{|J(\bar{\varkappa}_F - \bar{\varkappa}')|^2}{\hbar^2} \left[1 - \frac{\bar{v}(\bar{\varkappa}_F) \cdot \bar{v}(\bar{\varkappa}')}{|\bar{v}(\bar{\varkappa}_F)|^2} \right]$$
$$\cdot \langle \bar{s}(\bar{\varkappa} - \bar{\varkappa}') \cdot \bar{s}(\bar{\varkappa}' - \bar{\varkappa}) \rangle \tfrac{1}{2} [g(\bar{\varkappa}', 0) + g(\bar{\varkappa}', -\omega)] .$$

Now, if $\omega \lesssim \sigma_0$, the frequency dependence of γ^t may be neglected and we get Drude's law. However, if $\omega \gg \sigma_0$, the last term in (33) decreases the value of the integral, since $\bar{\varkappa}'$ is now compelled to be on an energy shell, of width $\hbar\sigma_0$ around the surface $\mu - \hbar\omega$. Moreover, near the ferromagnetic transition, $|\bar{\varkappa} - \bar{\varkappa}'| \lesssim k$ and thus, the last term will only pick-up spin correlations outside the critical region when $\hbar\omega/\mu \gg k/\varkappa_F$. We thus expect to obtain deviations from Drude's law [12] which are larger for higher frequencies or near the transition.

Appendix: Evaluation of the frequency-dependent conductivity.

From eqs. (1), (22) and (25) we get

$$\sigma_{\mu\nu}(\omega) = \frac{i e^2}{\hbar} \sum_\lambda \int \frac{d\bar{\varkappa}}{(2\pi)^3} v_\mu(\bar{\varkappa}) v_\nu(\bar{\varkappa}) \tag{A.1}$$
$$\cdot \left[\frac{1}{\beta\hbar} \sum_{\omega_n} \frac{F_\lambda(\bar{\varkappa}, \omega_n, \nu_n) - F_\lambda(\bar{\varkappa}, \omega_n, 0)}{i\nu_n} \right]_{i\nu_n = \omega + i\eta} ,$$

where

$$F_\lambda(\bar{\varkappa}, \omega_n, \nu_n)$$
$$= \Omega_\lambda(\bar{\varkappa}, \omega_n, \nu_n) / (1 - A_\lambda(\bar{\varkappa}, \omega_n, \nu_n)) \tag{A.2}$$

with Ω_λ and A_λ given, respectively, by (18) and (24). We compute the frequency summation in (A.1) by the usual complex integral:

$$\frac{1}{\beta\hbar} \sum_{\omega_n} F_\lambda(\bar{\varkappa}, \omega_n, \nu_n)$$
$$= -\frac{1}{2\pi i} \oint dz \, n(z) F_\lambda(\bar{\varkappa}, z, \nu_n) ,$$

where the contour surrounds the imaginary axis and $n(z)$ is the Fermi distribution. Now the contour may be deformed to surround the two branch cuts of function $F_{\lambda'}$ (the real axis and $z = -i\nu_n$). After performing the analytic continuation required in

(A.1), we get:

$$
A_\lambda(\bar{\varkappa}, z = \omega' \pm i0^+, i\nu_n = \omega + i\eta)
$$
$$
= \sum_{\lambda'} \int \frac{d\bar{\varkappa}'}{(2\pi)^3} v_{\lambda\lambda'}(\bar{\varkappa} - \bar{\varkappa}') \frac{\bar{v} \cdot \bar{v}'}{|\bar{v}|^2}
$$
$$
\cdot \frac{G_{\lambda'}^\pm(\bar{\varkappa}', \omega') - G_{\lambda'}^+(\bar{\varkappa}', \omega' + \omega')}{\omega + \frac{i}{2}\sigma_{\lambda'}^t(\bar{\varkappa}', \omega' + \omega) + \frac{i}{2}\sigma_{\lambda'}^t(\bar{\varkappa}', \omega')},
$$

where $G_\lambda^\pm(\bar{\varkappa}.\omega)$ are the advanced and retarded Green's function:

$$
G_\lambda^\pm(\bar{\varkappa}, \omega) = 1\bigg/\bigg(\omega - \omega_\lambda(\bar{\varkappa}) \pm \frac{i}{2}\sigma_\lambda^t(\bar{\varkappa}, \omega)\bigg). \quad (A.3)
$$

We have neglected the real part of the self-energy and denoted by σ^t the total contribution to the self-energy. First consider the function

$$
A_\lambda(\bar{\varkappa}, z = \omega' - i0^+, i\nu_n = \omega + i\eta).
$$

The function $G_{\lambda'}^-(\bar{\varkappa}', \omega') - G_{\lambda'}^+(\bar{\varkappa}', \omega' + \omega)$ has an imaginary part equal to

$$
\frac{i}{2}(g_{\lambda'}(\bar{\varkappa}', \omega') + g_{\lambda'}(\bar{\varkappa}', \omega' + \omega)).
$$

On the other hand its real part (which vanishes for $\omega = 0$) is the difference of two functions which change sign and vary very rapidly at $\omega_{\lambda'}(\bar{\varkappa}') = \omega'$ and $\omega_{\lambda'}(\bar{\varkappa}') = \omega' + \omega$, respectively. The range of variation is $o(\sigma_\lambda)$; therefore, if we transform the wave vector integral to an integral over $\omega_{\lambda'}$, and if the remaining integrand changes slowly over this range, the integral is approximately zero.

The same type of arguments leads to

$$
A_\lambda(\bar{\varkappa}, z = \omega' \pm i0^+, i\nu_n = 0) \approx 0.
$$

Therefore, we get:

$$
A_\lambda(\bar{\varkappa}, z = \omega' - i0^+, i\nu_\lambda = \omega + i\eta) \quad (A.4)
$$
$$
\cong \sum_{\lambda'} \int \frac{d\bar{\varkappa}'}{(2\pi)^3} v_{\lambda\lambda'}(\bar{\varkappa} - \bar{\varkappa}') \frac{\bar{v} \cdot \bar{v}'}{|\bar{v}|^2}
$$
$$
\cdot \frac{\frac{1}{2}[g_{\lambda'}(\bar{\varkappa}', \omega') + g_{\lambda'}(\bar{\varkappa}', \omega' + \omega)]}{-i\omega + \frac{1}{2}\sigma_{\lambda'}^t(\bar{\varkappa}', \omega') + \frac{1}{2}\sigma_{\lambda'}^t(\bar{\varkappa}', \omega' + \omega)}.
$$

The 2nd member is just the function

$$
\eta_\lambda(\bar{\varkappa}, \omega' + \omega, \omega),
$$

see (28). Now consider the case

$$
A_\lambda(\bar{\varkappa}, z = \omega' + i0^+, i\nu_n = \omega + i\eta).
$$

Transforming again the wave-vector integral into an integration over $\omega_{\lambda'}$, we see that the function $G_{\lambda'}^+(\bar{\varkappa}', \omega_l) - G_{\lambda'}^+(\bar{\varkappa}', \omega' + \omega)$ has poles on the same half of the complex plane. Thus, if the rest the the integrand is an analytic function of $\omega_{\lambda'}$, we just get:

$$
A_\lambda(\bar{\varkappa}, z = \omega' + i0^+, i\nu_n
$$
$$
= \omega + i\eta) \approx 0. \quad (A.5)
$$

By the same reasoning,

$$
A_\lambda(\bar{\varkappa}, z = -i\nu_n + \omega' + i0^+, i\nu_n = \omega + i\eta)
$$
$$
\approx \sum_{\lambda'} \int \frac{d\bar{\varkappa}'}{(2\pi)^3} v(\bar{\varkappa} - \bar{\varkappa}') \frac{\bar{v} \cdot \bar{v}'}{|\bar{v}|^2}
$$
$$
\cdot \frac{\frac{1}{2}[g_{\lambda'}(\bar{\varkappa}', \omega' - \omega) + g_{\lambda'}(\bar{\varkappa}', \omega')]}{-i\omega + \frac{1}{2}\sigma_{\lambda'}^t(\bar{\varkappa}', \omega' - \omega) + \frac{1}{2}\sigma_{\lambda'}^t(\varkappa', \omega')}
$$
$$
= \eta_\lambda(\bar{\varkappa}, \omega', \omega), \quad (A.6)
$$
$$
A_\lambda(\bar{\varkappa}, z = -i\nu_n + \omega' - i0^+, i\nu_n = \omega + i\eta) \approx 0.
$$

Using these results, we get

$$
\left[\frac{1}{\beta\hbar}\sum_{\omega_n} F_\lambda(\bar{\varkappa}, \omega_n, \nu_n) - F_\lambda(\bar{\varkappa}, \omega_n, 0)\right]_{i\nu_n = \omega + i\eta} = -\frac{1}{2\pi i}
$$
$$
\cdot \int d\omega'\, n(\omega')[G_\lambda^+(\bar{\varkappa}, \omega')(G_\lambda^+(\bar{\varkappa}, \omega' + \omega) - G_\lambda^+(\bar{\varkappa}, \omega')(-G_\lambda^-(\bar{\varkappa}, \omega' - \omega)G_\lambda^-(\bar{\varkappa}, \omega')]
$$
$$
- \frac{1}{2\pi i}\int d\omega'\,(n(\omega') - n(\omega' - \omega))\frac{G_\lambda^-(\bar{\varkappa}, \omega' - \omega)G_\lambda^+(\bar{\varkappa}, \omega')}{1 - \eta_\lambda(\bar{\varkappa}, \omega', \omega)}. \quad (A.7)
$$

Now if we insert (A.7) into (A.1) and transform the wave vector integral into an integral over ω_λ, we see that the first term of (A.7) gives no contribution (if the remaining integrand is analytical). On the other hand,

$$
G_\lambda^-(\bar{\varkappa}, \omega' - \omega)G_\lambda^+(\bar{\varkappa}, \omega') = [G_\lambda^+(\bar{\varkappa}, \omega') - G_\lambda^-(\bar{\varkappa}, \omega' - \omega)]\bigg/\bigg[-\omega - \frac{i}{2}\sigma_\lambda^t(\bar{\varkappa}, \omega' - \omega) - \frac{i}{2}\sigma_\lambda^t(\bar{\varkappa}, \omega')\bigg].
$$

Again, we separate the difference between the two Green's functions into its real and imaginary parts. Repeating the reasoning that lead to (A.4), we just keep the imaginary part and get

$$G_\lambda^-(\bar{\varkappa}, \omega' - \omega) G_\lambda^+(\bar{\varkappa}, \omega') \cong \tfrac{1}{2}[g_\lambda(\bar{\varkappa}, \omega') + g_\lambda(\bar{\varkappa}, \omega' - \omega)]/[-i\omega + \tfrac{1}{2}\sigma_\lambda^t(\bar{\varkappa}, \omega') + \tfrac{1}{2}\sigma_\lambda^t(\bar{\varkappa}, \omega' - \omega)].$$

Putting everything together, we finally obtain

$$\sigma_{\mu\nu}(\omega) = \frac{e^2}{\hbar} \sum_\lambda \int \frac{d\omega'}{2\pi} \frac{n(\omega' - \omega) - n(\omega')}{\omega}$$
$$\cdot \int \frac{d\bar{\varkappa}}{(2\pi)^3} \frac{v_\mu(\bar{\varkappa}) v_\nu(\bar{\varkappa})}{1 - \eta_\lambda(\bar{\varkappa}, \omega', \omega)} \frac{\tfrac{1}{2}[g_\lambda(\bar{\varkappa}, \omega' - \omega) + g_\lambda(\bar{\varkappa}, \omega'))}{-i\omega + \tfrac{1}{2}\sigma_\lambda^t(\bar{\varkappa}, \omega' - \omega) + \tfrac{1}{2}\sigma_\lambda^t(\bar{\varkappa}, \omega')}.$$

Acknowledgements

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